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Altering the Fibonacci formula for rabbit Farm

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Abstract
We have a Fibonacci recurrence relation $F_k = F_{k-1} + F_{k-2}$ with initial conditions $F_0 = 1$ and $F_1 = 1$. This recurrence relation can be derived by considering a rabbit production case. While deriving this recurrence relation from the rabbit production case, we suppose that rabbits don't die. We also suppose that there is only one pair at start. This recurrence relation doesn't look suitable to be applied in a real world case, because rabbits may die or be sold for meat etc. Moreover, in a real world rabbit farm, initial pairs are more than one. In this paper, we will produce a new recurrence relation by altering and modifying the current recurrence relation. We will incorporate the fact that rabbits may die or be sold. The new recurrence relation would be more suitable and feasible for practical cases.

Keywords*:* Probability, ceiling, recurrence relation.

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Introduction

Our objective in this paper is to produce a new recurrence relation from the present Fibonacci recurrence relation. The reason for producing new recurrence relation is justified in the following lines. Present Fibonacci recurrence relation is $F_k = F_{k-1} + F_{k-2}$ with initial conditions $F_0 = 1$ and $F_1 = 1$. We can derive this recurrence relation (formula) by considering a rabbit production case. While deriving this formula from rabbit production case, we suppose that rabbits don't die and also that there is only one pair at start. But in a real world farm, rabbits do die and be sold for meat, and also there are many pairs of rabbits at start. So the present formula is not suitable to be applied on a real world rabbit farm. In this paper we will produce a new recurrence relation by altering and modifying the current recurrence relation. We will incorporate into the present formula the fact that rabbits may die or be sold. The new recurrence relation would be more suitable and feasible for application at rabbit farm.

Organization of the paper as follows. In section II, we will review the derivation of Fibonacci formula and discuss its limitations in details. In section III, we will start proposing new solution and generalize this formula. In section IV, we will alter this formula by supposing that the rabbits may die or be sold. That would be our new final formula. In section V, we will give an algorithm for final formula. In section VI, we will provide a C++ program to show an application of our new formula. In section VII, we will conclude our discussion.

Review and related work

In this section we will review the derivation of present Fibonacci formula and discuss its limitations. After that, we will provide the reasons for proposing our new formula. Let us start now.

Suppose we have a rabbit farm. We have a pair of rabbits. After two months, this pair will produce a new pair every month. The newly born pair, after two months, will also be able to produce another new pair every month. This process will continue in the same way and we will have many pairs of rabbits after some months. Suppose no pair of rabbits dies; then the production of pairs of rabbits will progress something like shown in Table I.

Table 1: Rabbit production

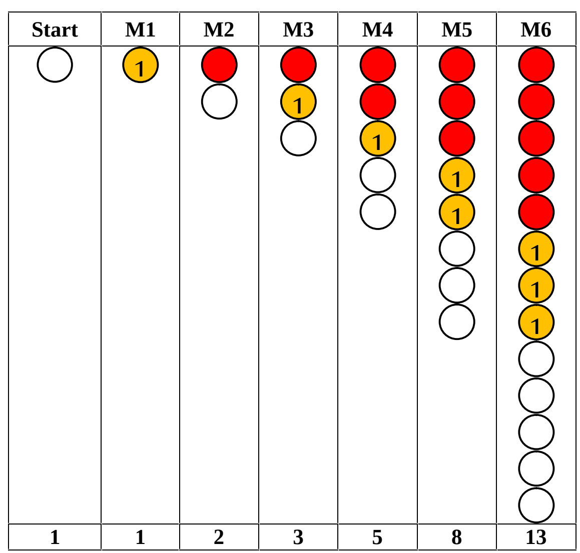

Table I shows the month-wise counting of rabbit production. The symbol shows the newly born pair. The symbol shows a one month old pair. The symbol shows a pair with age greater than or equal to two months.

Production Rules:

- Every newly born pair will be converted to one month old pair in the next month. It will not produce any new pair
- Every one month old pair will be converted to two months old pair in the next month and it will produce a new pair
- Every two or greater than two months old pair will be converted to two or greater than two months old pair in the next month and will produce a new pair

By applying the production rules to table I, we have calculated the number of pairs of rabbits from start to month-6 and these are 1, 1, 2, 3, 5, 8, and 13

Let us make an observation on table I. We want to calculate the number of pairs for month-7. We know that all the pairs at month-6 will also be available at month-7. We also know that all of the pairs at month-6 will not be able to produce new pairs at month-7 because some pairs at month-6 are newly born. But if we give a thoughtful look, we can see that all the pairs at month-5 will be greater than or equal to two months of age at month-7, and will produce new pairs at month-7. So in a conclusion, the number of pairs at month-7 will be equal to the sum of pairs at month-6 and month-5. In general, the number of pairs at any month would be equal to the sum of pairs at previous month and previous of previous month. In this way we can calculate the number of pairs at any month. Now the counting of pairs of rabbits up to some months will be as 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, and so on. This gives us our Fibonacci formula $F_k = F_{k-1} + F_{k-2}$ with initial conditions $F_0 = 1$ and $F_1 = 1$.

Now let us discuss the limitations of this Fibonacci formula due to which we cannot apply it on our real world rabbit farm or any other animal farm. Limitations are:

- We suppose in the formula that rabbits don't die. Whereas rabbits do die and may be sold for meat. So this formula gives us a larger number of rabbits than the actual counting.
- We suppose in the formula that there is only one pair at start. Whereas, in actual, the pairs may be much more than one.
- We suppose that every new pair produces a new pair after two months. Whereas, in actual, some animals become able to produce new pair after several months.
- We suppose that once a pair is able to produce a new pair, it produces a new pair every month afterwards. Whereas, in actual, this duration is different for different animals.
- We suppose that a pair produces one new pair. Whereas, in actual, some animal may produce more than one pair.

This was all about the Fibonacci formula and its limitations for our rabbit farm. Now in coming sections of this paper, we will alter and modify this formula to produce our new formula which will handle all the limitations discussed.

Fibonacci formula generalized

Our Fibonacci formula so far is $F_k = F_{k-1} + F_{k-2}$. Here all the pairs of rabbits at stage F_{k-1} will also be present at stage F_k. There are also some pairs at stage F_{k-1} that will produce new pairs at stage F_k. These pairs are, in fact, all those pairs that are 2 or greater than 2 months old. All such pairs are in fact those pairs that are present at stage F_{k-2}. Now suppose that after 2 months, instead of producing 1 pair each month, a pair produces 2 new pairs, or 3 new pairs, or m new pairs. In this case the Fibonacci formula will look like $F_k = F_{k-1} + 2F_{k-2}$, $F_k = F_{k-1} + 3F_{k-2}$, $F_k = F_{k-1} + mF_{k-2}$ respectively.

Now suppose again that a pair produces m new pairs not after two months, but after 3 months. In this case, all the pairs at stage F_{k-1} will also be present at stage F_k. There are also some pairs at stage F_{k-1} that will produce new pairs. These pairs are all those pairs that are 3 or greater than 3 months old. All such pairs are in fact those pairs that are present at stage F_{k-3}. Now suppose that a pair produces m new pairs after 3 months, 4 months or n months. In this case the Fibonacci formula will look like $F_k = F_{k-1} + mF_{k-3}$, $F_k = F_{k-1} + mF_{k-4}$, $F_k = F_{k-1} + mF_{k-n}$ respectively.

So the generalized form of Fibonacci formula can be written as $F_k = F_{k-1} + mF_{k-n}$ where m is the number of pairs of rabbits that a pair produces, and n is the number of months after which rabbits start producing m new pairs every month.

Generalized Fibonacci formula with death and sale probability incorporated

We have, so far, produced the generalized Fibonacci formula $F_k = F_{k-1} + mF_{k-n}$. But this formula is still not suitable or feasible to be applied at our Rabbit Farm. Why? Because in this formula we still have the supposition that rabbits don't die. So this supposition will count a larger number of rabbits than that of the original number because in real world case, rabbits do die and sold for meat as well. It means all the pairs present at stage F_{k-1} will not be present at stage F_k and all the pairs at stage F_{k-n} will not produce new pairs at stage F_k due to the probability that some rabbits may die or be sold in these durations. Now we will produce a new formula that will incorporate in it, the death and sale probability of rabbits.

Let P_1 is the probability of dead and sold pairs of stage F_{k-1} within 1 month duration from stage F_{k-1} to stage F_k . The probability P_1 can be shown as:

$$P_1 = \frac{\text{Dead+Sold Pairs of stage } \mathrm{F}_{\text{k-1}}}{\text{Total pairs at stage } \mathrm{F}_{\text{k-1}}}, \text{ where } 0 \le P_1 \le$$

Now the number of dead and sold pairs of stage F_{k-1} from F_{k-1} to F_k is equal to $\lceil P_1(F_{k-1}) \rceil$. Here ceiling $\lceil \ \rceil$ is taken to get an integer value. Thus as a result, the number of pairs that will be shifted from stage F_{k-1} to F_k would be equal to $F_{k-1} - \lceil P_1(F_{k-1}) \rceil$

We also know that the pairs of rabbits at stage F_{k-1} which are ready to produce new rabbits at stage F_k are, in fact, those pairs that were present at stage F_{k-n}. Here again we know that some pairs of stage F_{k-n} may also die or be sold. Let P_n is the probability of dead and sold pairs of stage F_{k-n} within *n* month duration from stage F_{k-n} to the stage F_k. The probability P_n can be shown as:

$$P_n = \frac{\text{Dead+Sold pairs of stage } \mathrm{F}_{\text{k-n}}}{\text{Total pairs at stage } \mathrm{F}_{\text{k-n}}}, \text{ where } 0 \le P_n \le 1$$

Now the number of dead and sold pairs of stage F_{k-n} from F_{k-n} to F_k is equal to $\lceil P_n(F_{k-n}) \rceil$. The ceiling $\lceil \ \rceil$ is taken to get an integer value. Thus as a result, the number of pairs that will produce new pairs for stage F_k would be equal to $F_{k-n} - \lceil P_n(F_{k-n}) \rceil$. So the newly produced rabbits for F_k would be $m\left(F_{k-n} - \lceil P_n(F_{k-n}) \rceil\right)$.

Now the total number of rabbits at stage F_k would be

$$F_k = \left(F_{k-1} - \lceil P_1(F_{k-1}) \rceil\right) + m\left(F_{k-n} - \lceil P_n(F_{k-n}) \rceil\right)$$

and this is our required altered version of Fibonacci recurrence formula which is more feasible and suitable to be applied at our Rabbit Farm. An interesting thing to note here is that we can still use this altered formula as an original formula by setting the values of P_1 and P_n to 0.

Algorithm for altered Fibonacci formula

Here is the algorithm that will use the new formula to count the pairs of rabbits up to any given duration.

CountPairs(initialPairs, p1, pn, newPairsPerPair, monthsForFirstProduction, lastMonth)

1. $P_1 = \text{p1}$, $P_n = \text{pn}$, $m = \text{newPairsPerPair}$, $n = \text{monthsForFirstProduction}$,
2. $F = \text{new array[lastMonth]}$; $F[0] = \text{initialPairs}$;
3. For k=1 to n-1
   do $F[k] = \left(F[k-1] - \lceil P_1(F[k-1]) \rceil\right)$
4. For k=n to lastMonth
   do $F[k] = \left(F[k-1] - \lceil P_1(F[k-1]) \rceil\right) + m\left(F[k-n] - \lceil P_n(F[k-n]) \rceil\right)$
5. Output F;

This is a generic algorithm that, now using little trick, can be used to count any type of animals up to any duration.

An application for rabbit farm

Here we are creating a small and simple C++ application by using our altered version of Fibonacci formula. The application will count rabbits or other animals up to any duration. Following is the C++ application code.

```
#include <iostream.h>
#include <conio.h>
#include <math.h>
main(){
  cout<< "Please enter:\n";
  cout<<"Probability P1: ";
  float p1 = 0.0;     cin >> p1;

  cout<<"Probability Pn: ";
  float pn = 0.0;     cin >> pn;

  cout<<"Number of initial pairs: ";
  int initialPairs = 0;   cin >> initialPairs;

  cout<<"New pairs that a pair will produce: ";
  int m = 0;    cin >> m;

  cout<<"The last month: ";
  int lastMonth = 0;    cin >> lastMonth;
  lastMonth += 1;

  cout<<"No. of months for first production: ";
  int n = 0;    cin >> n;

  int F[lastMonth];
  F[0]= initialPairs;

  for(int k=1; k<=n-1; k++)
        { F[k]= F[k-1] - ceil(p1*F[k-1]); }

  for(int k = n; k<lastMonth; k++)
       {  F[k] = (F[k-1] - ceil(p1*F[k-1])) + m*(F[k-n] -
ceil(p1*F[k-n])); }

  for( int k = 0; k < lastMonth; k++)
      {  cout<< "M" << k << "\t = \t" << F[k] << "\n"; }
    getch();
}
```

The above code has been compiled and run to solve 2 examples. The snapshots of application have been given next.

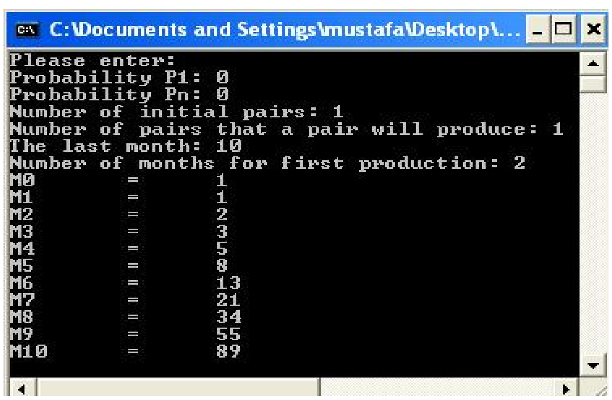

Fig. 1: Generating Fibonacci sequence (Example-1)

Fig. 1 shows the snapshot of application when the application was used to count some terms of Fibonacci sequence (Example-1). We have set P_1 and P_n to 0 here, because we suppose that rabbits don't die in original Fibonacci formula. The altered Fibonacci formula works similar to original formula when we set P_1 and P_n to 0.

Fig. 2 shows the snapshot of application when it was used to solve Example-2. In this example we have supposed that rabbits may die or be sold. Here we have set P_1 to 0.1 and P_n to 0.3. Other inputs have also been changed.

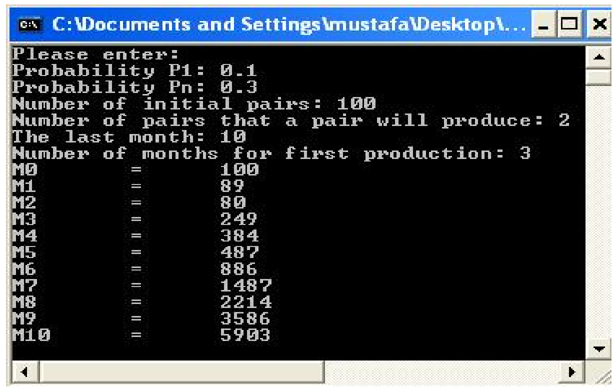

Fig. 2: Application of altered Fibonacci formula (Example-2)

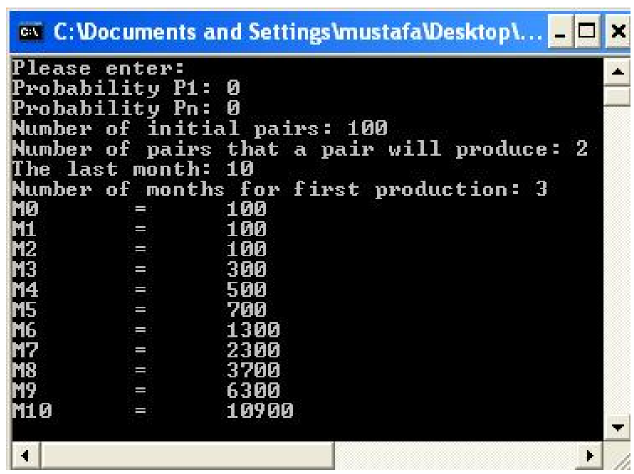

Fig. 3: Application of original Fibonacci formula (Example-2)

Now let us run the application to solve example-2 again. But this time we will suppose that rabbits don't die. So we will set P_1 and P_n to 0. Here again our alter Fibonacci formula will work similar to original formula. Snapshot of the application has been given in Fig. 3. We can observe that when example-2 was solved using altered version of Fibonacci formula, total pairs after 10 months were 5903 (Fig. 2), and when the same example was solved using the original Fibonacci formula, total pairs after 10 months were 10900 (Fig. 3). 5903 is very much closer to real world total because here we have considered that rabbits do die and be sold. 10900 is a wrong total because here we suppose that rabbits don't die. And that is the difference between counting which motivated us to produce our altered version of Fibonacci formula.

Conclusions

In this paper we have seen that the original Fibonacci formula cannot be applied at our real world rabbit farm due to its theoretic-concepts. It supposes that rabbits don't die. So, at any stage, it calculates a total that is many times larger than the original total. It is also not flexible to be applied for counting of other animals. We produced an altered Fibonacci formula that had practical-concepts.

New formula incorporates in it, the probability that rabbits may die and be sold. New formula guesses a total that is much closer to original total. So it is more suitable to be applied at our rabbit farm. New formula can also be applied for other animals having different production rates and durations. In future, we will attempt to give a solution to our new formula for *nth* term.

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